Ultra High-Precision Astrometry with the Habitable Worlds Observatory

Scott Gaudi

The Ohio State University

With contributions from Eduardo Bendek, Kaz Gary (OSU Graduate Student), Aki Roberge, Breann Starski

A Future Space Mission with Very High Precision Astrometry Workshop

Institut d'Astrophysique de Paris

9/12/2p024

National Aeronautics and Space Administration

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Warning: work in progress

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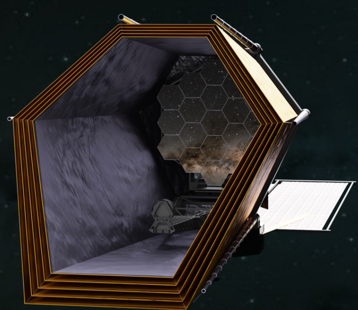
Preliminary specs & candidate instruments

Telescope

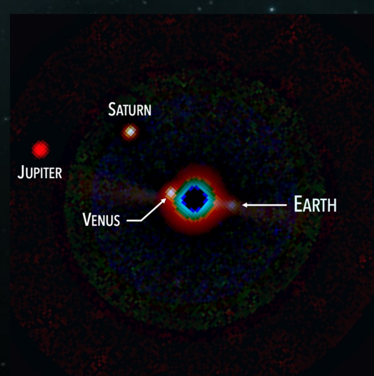
Diameter	~6.0 m (inner)
Bandpass	~100–2500 nm



Coronagraph

High-contrast imaging and imaging spectroscopy

Bandpass	~200–1800 nm
Contrast	$\lesssim 1 \times 10^{-10}$
R ($\lambda/\Delta\lambda$)	Vis: ~140 NIR: ~70, 200



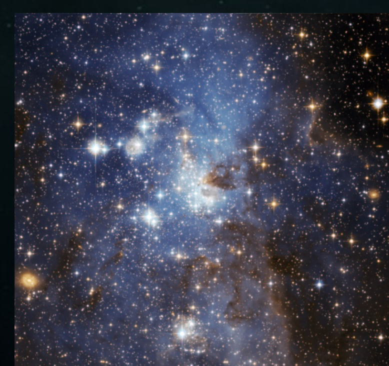
High-Resolution Imager

UV/Vis and NIR imaging

Bandpass	~200–2500 nm
Field-of-View	~3′ × 2′

60+ science filters & grism

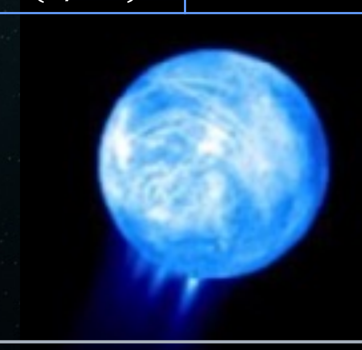
High-precision astrometry?



UV Multi-Object Spectrograph

UV/Vis multi-object spectroscopy and FUV imaging

Bandpass	~100–1000 nm
Field-of-View	~2′ × 2′
Apertures	~840 × 420
R ($\lambda/\Delta\lambda$)	~500–60,000



Fourth Instrument

To be defined

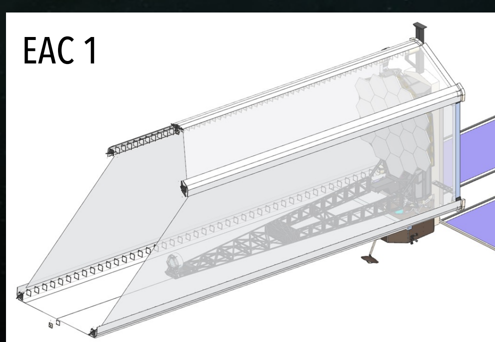
+ Two guide cameras

Technical Activities – Exploratory Analytic Cases (EACs)

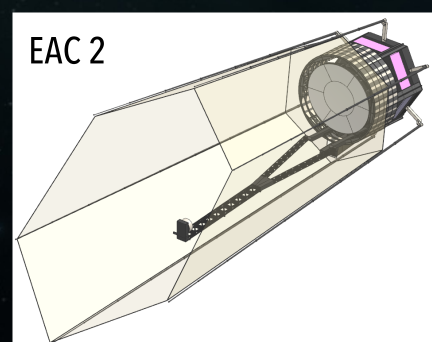
These are 1st round mission architectures that will be used to explore the HWO trade space. Purposes …

- Practice end-to-end modeling, from science to engineering. Develop initial codes to "pipeclean" the process
- Use EACs to identify key technology gaps and guide maturation of potential technology solutions
- Provide feedback to rocket vendors as soon as possible to help influence their direction

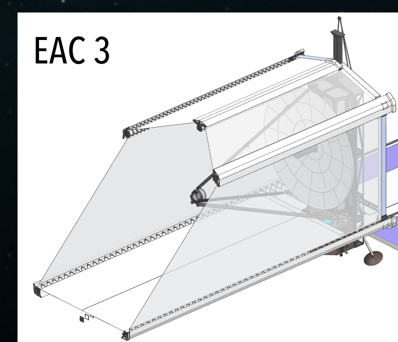
Exploration of three Round 1 EACs will take ~ 1 year. Findings will fold into Round 2 EACs.



6-m inner diameter / 7.2-m outer diameter off-axis



6-m diameter off-axis

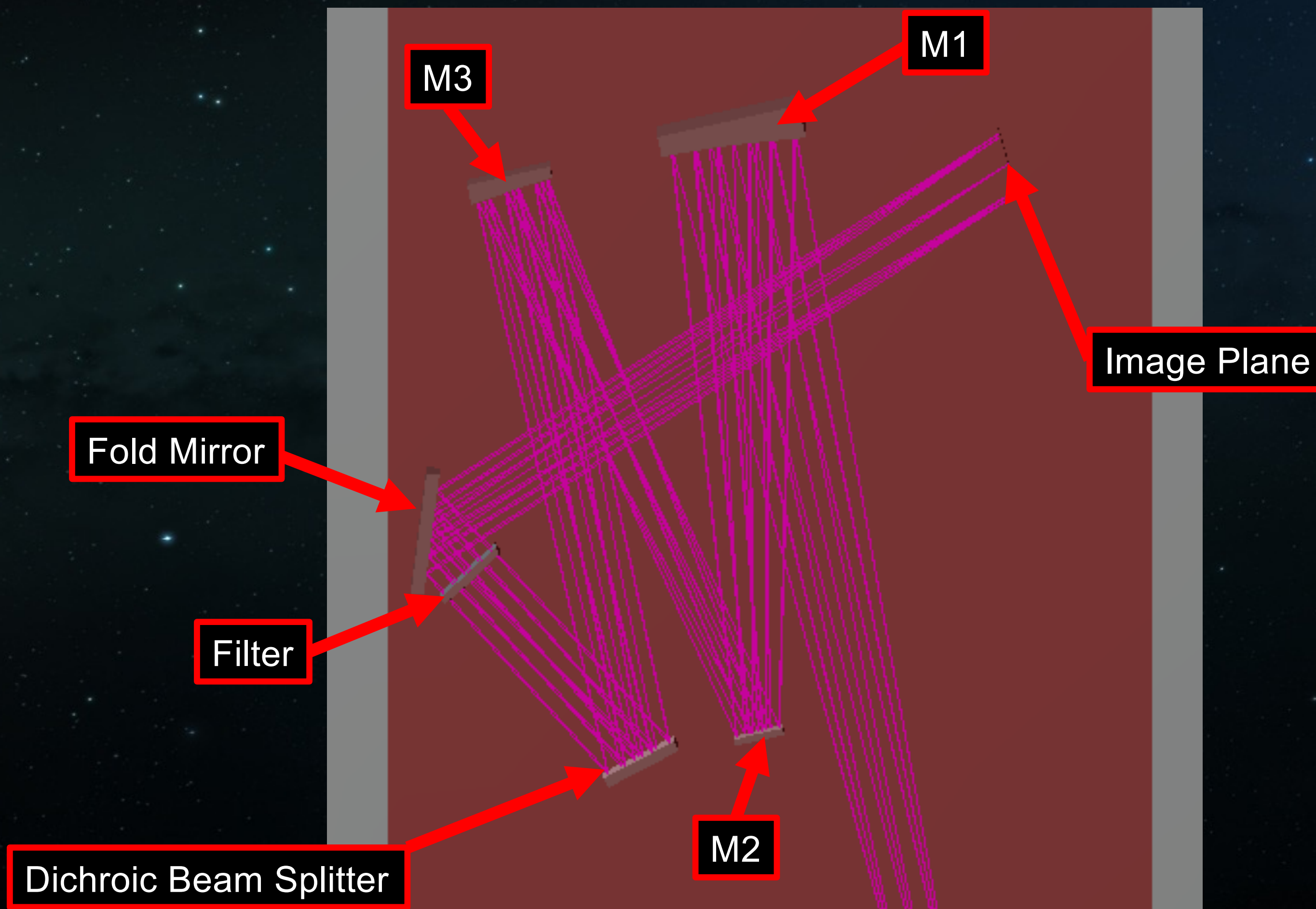


8-m diameter on-axis

High Resolution Imager

High-Resolution Imager	
UV/Vis and NIR imaging	
Bandpass	~200–2500 nm
Field-of-View	~3′× 2′
UVIS Camera	CMOS
	Nyquist @ 500nm
	8.60 mas/pix
	~300 Mpix camera
	Four 8k×8k chips
NIR Camera	HgCdTe
	Nyquist @ 1000nm
	17.29 mas/pix
	~72 Mpix camera
	Four 4k×4k chips

EAC1 HRI UVIS Instrument Layout



High Resolution Imager

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	Four 8k×8k chips
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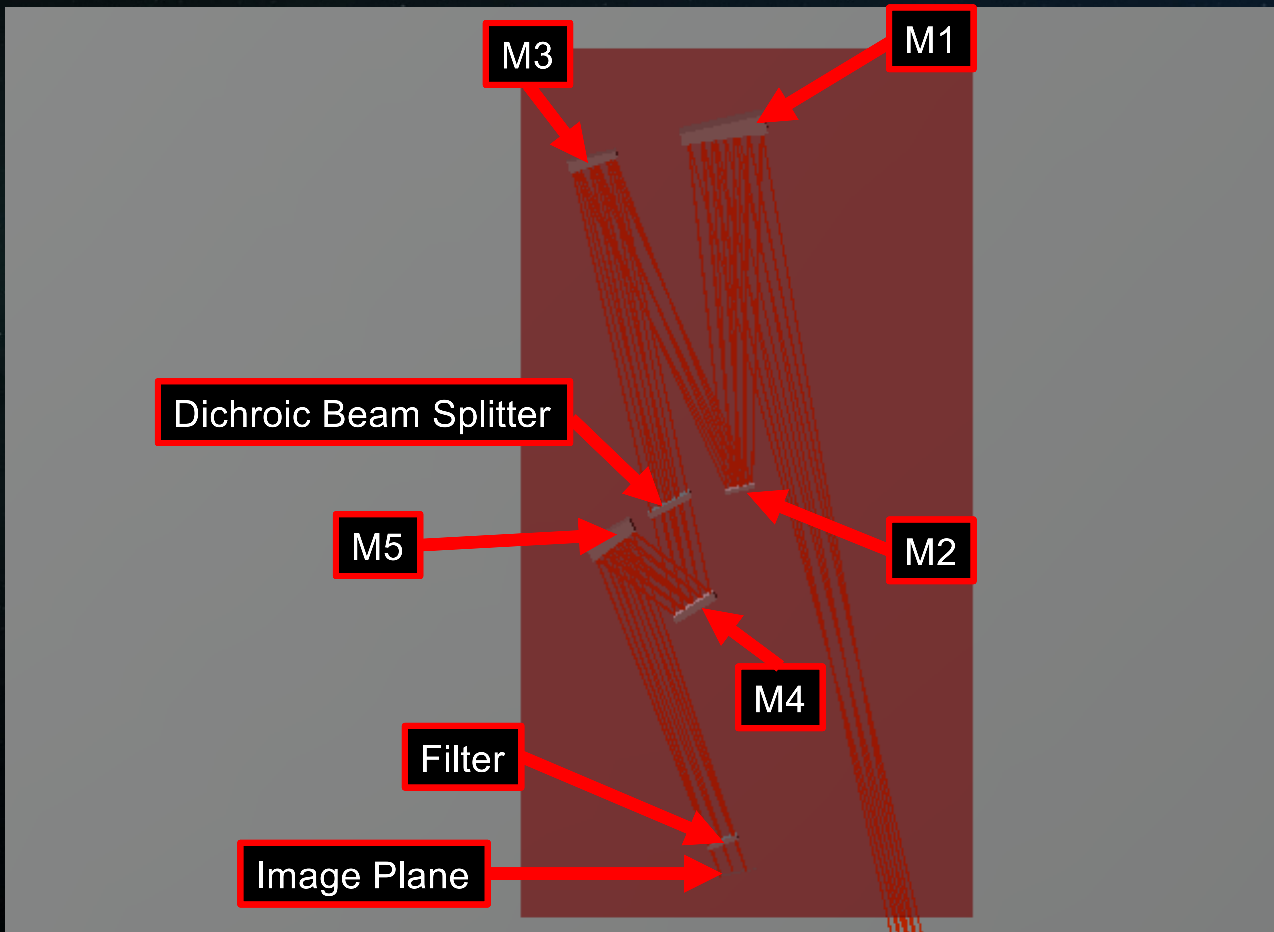
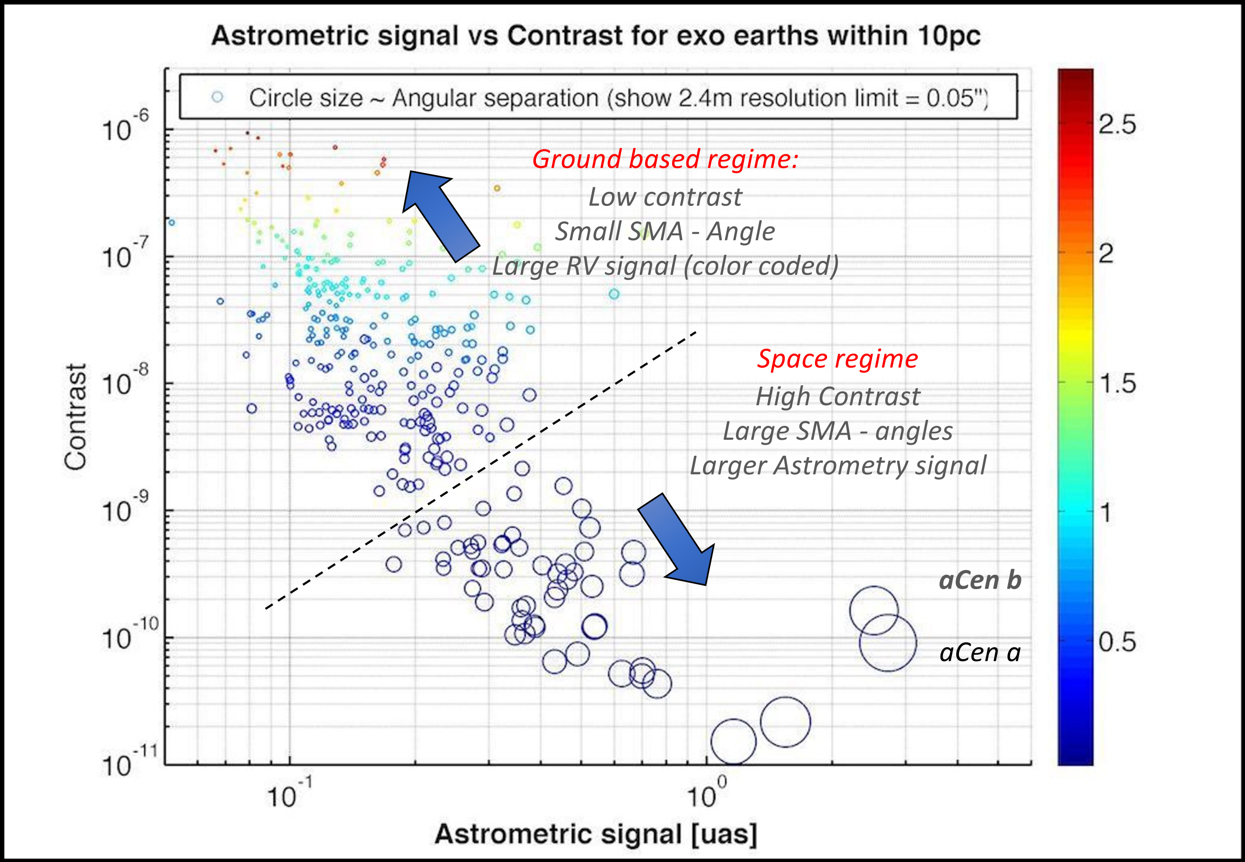
	Nyquist @ 1000nm
	17.29 mas/pix
	~72 Mpix camera
	Four 4k×4k chips

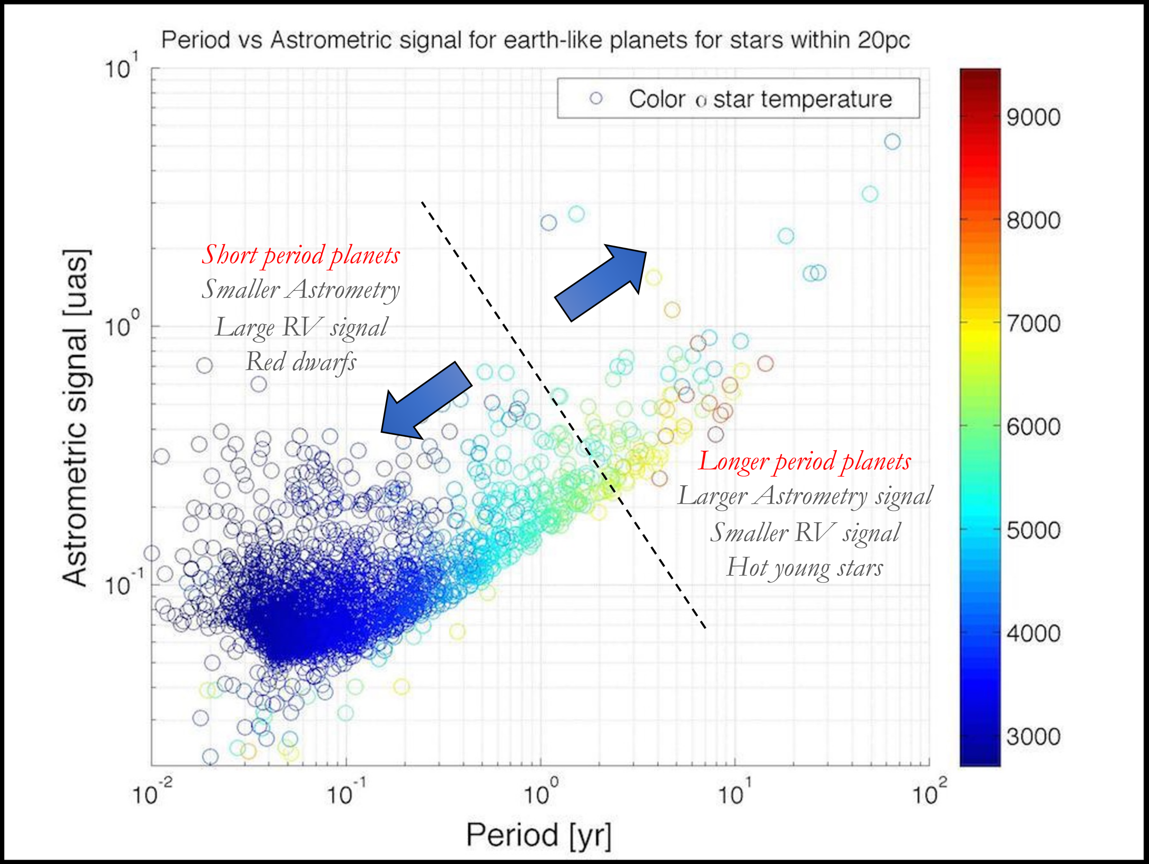
EAC1 HRI NIR Instrument Layout



Masses of Nearby Habitable Planets: Astrometry, Direct Imaging and RV

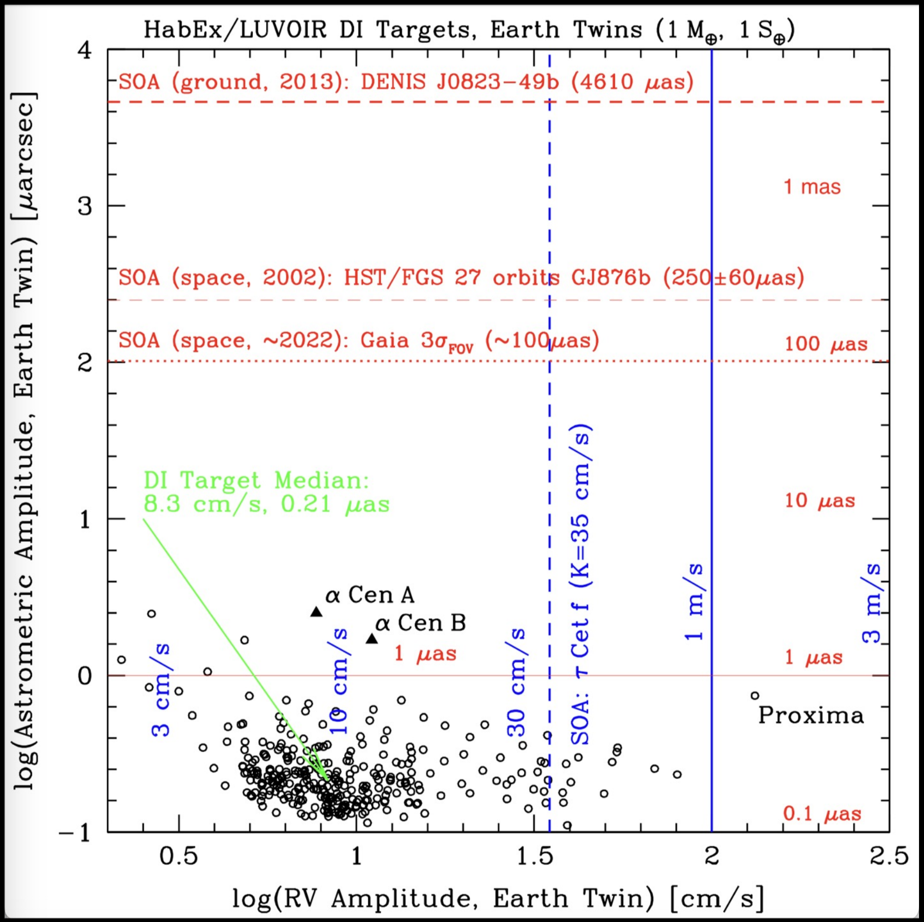


Courtesy of Eduardo Bendek



Courtesy of Eduardo Bendek

Astrometry, Direct Imaging and RV



Plot by Eric Mamajek

Astrometric Detection of Exoplanets

- This approximate ${}^{S}/_{N}$ assumes *N* uniform-sampled 2D astrometric measurements each with precision σ along each axis,.
- The astrometric signal of planets with periods longer than the duration of the survey T are very hard to detect
- The signals are mostly linear 'trends', which cannot be distinguished from the host star proper motion.

$$
{}^{S}/_{N} = \begin{cases} \sqrt{N} F(i, e, \omega) \dfrac{\alpha_0}{\sigma} & \text{if } P \lesssim T \\ 0 \text{ if } P \gtrsim T \end{cases},
$$

where $\alpha_0 \equiv \left(\frac{a}{d}\right)\left(\frac{m_p}{M_*}\right)$ is the astrometric signal for a circular, face-on orbit, and

$$
F(i, e, \omega) = \sqrt{\frac{1}{2}(1 + \cos^2 i)\left[1 - e^2\left[3 - \left(\frac{2}{1+\cos^2 i} - 1\right)\right]\cos^2 \omega\right]}
$$

accounts for the effects of inclination i, eccentricity e, and longitude of periastron ω, and is generally of order unity.

HZ planets: signal and comparison to RV

- The signal for an Earth analog at 10 pc is 0.3 μas.
- For HZ planets, assuming $L_* \propto M_*^4$, the signal increases as M_*^2.
- All else fixed, planets in the HZ are easier to detect around higher mass stars.
- For RV, the signal goes as $M_*^{-3/2}$, so HZ planets are easier to detect around lower-mass stars.
- High mass stars also have fewer spectral lines and ten to be rapidly rotating (above the Kraft break).

$$\alpha_0 \equiv 0.3\mu as \left(\frac{a}{AU}\right)\left(\frac{m_p}{M_\oplus}\right)\left(\frac{M_*}{M_\odot}\right)^{-1}\left(\frac{d}{10pc}\right)^{-1}$$

$$\alpha_0 \sim 0.3\mu\text{as}\left(\frac{m_p}{M_\oplus}\right)\left(\frac{M_*}{M_\odot}\right)\left(\frac{d}{10pc}\right)^{-1} \text{ for HZ}$$

$$K \sim 10\text{ cm/s}\left(\frac{m_p}{M_\oplus}\right)\left(\frac{M_*}{M_\odot}\right)^{-3/2} \text{ for HZ}$$

Detection and Mass Uncertainty

$$ {}^{\mathrm{S}}/_{\mathrm{N}} \simeq 10F\left(\frac{N}{100}\right)^{1/2}\left(\frac{\sigma}{0.3\mu\mathrm{as}}\right)^{-1}\left(\frac{m_p}{M_\oplus}\right)\left(\frac{M_*}{M_\odot}\right)^{2} \text{ if } P \lesssim T $$

- A planet m_p $=M_\oplus$ planet in the HZ can be detected with ${}^{\mathrm{S}}/_{\mathrm{N}} \simeq 10$ with N=100 (2D) astrometric measurements each with precision $\sigma = 0.3\ \mu\mathrm{as}$ along each axis.
- This corresponds to a 10% mass measurement if the observations are spread out uniformly in phase and the duration of observation is (much) greater than the period.
- Highly simplified treatment but provides a rough sense of the number and quality of the astrometric measurements needed.

Photon Noise Astrometric Uncertainty

$$\sigma_a \simeq 1.7\mu\text{as}\ 10^{-0.2(V_{ref}-15)} \left(\frac{t_{exp}}{300\text{s}}\right)^{-1/2} \left(\frac{D}{6.5m}\right)^{-1} \left(\frac{\lambda/D}{0.0159\ \text{as}}\right)$$

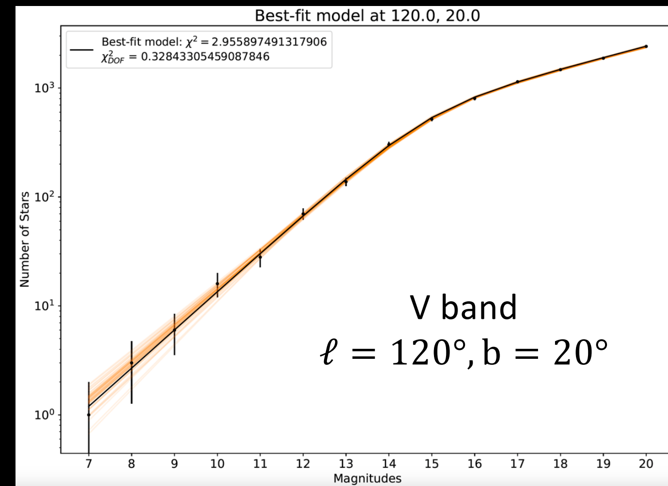
- The astrometric uncertainty for a $V_{ref} = 15$ reference star is ~1.7μas for a 5-minute exposure on HWO in the V band.
- Need ~30 reference stars with $V_{ref} \leq 15$ to achieve the total astrometric uncertainty of ~0.3 μas per epoch.

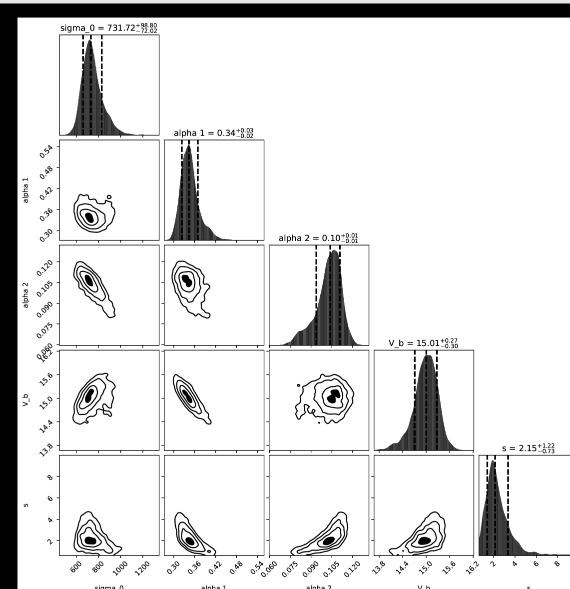
Number of reference stars

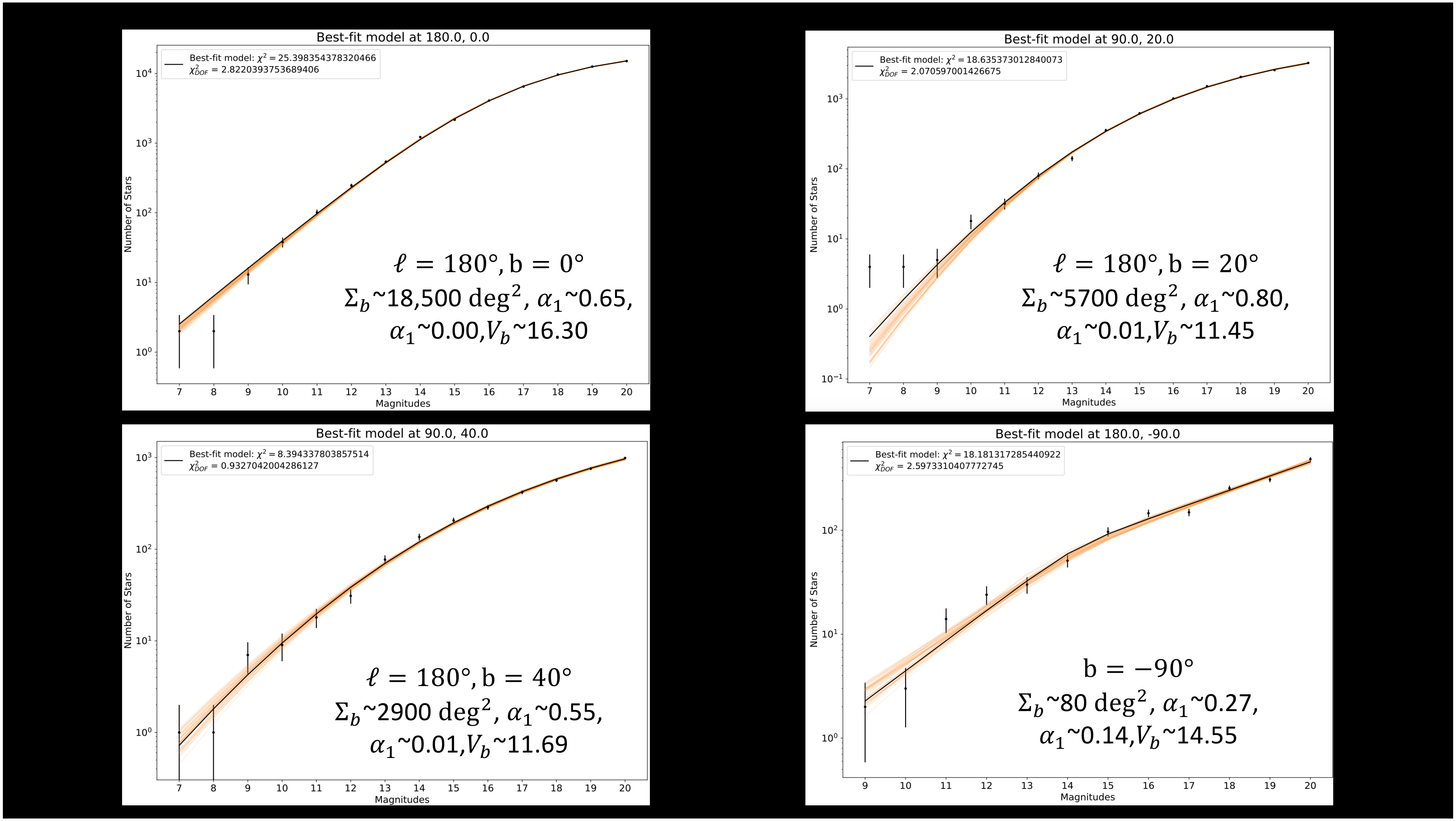
- For HWO, the number of reference stars in the FOV of the camera is a critical consideration
- Depends on Galactic latitude and longitude and filter
- Generally, follows a broken power law.

$$\Sigma(V) = \Sigma_b\left[10^{-s\alpha_1(V-V_b)} + 10^{-s\alpha_2(V-V_b)}\right]^{-1/s}$$

- α_1 is the bright-end slope, α_2 is the faint-end slope, Σ_b is the # stars/deg^2/mag at $V = V_b$, and s is the 'softness' of the break
- α_1~0.4 implies a uniform number of density of stars of a fixed magnitude
- Broken power law due to disk geometry.





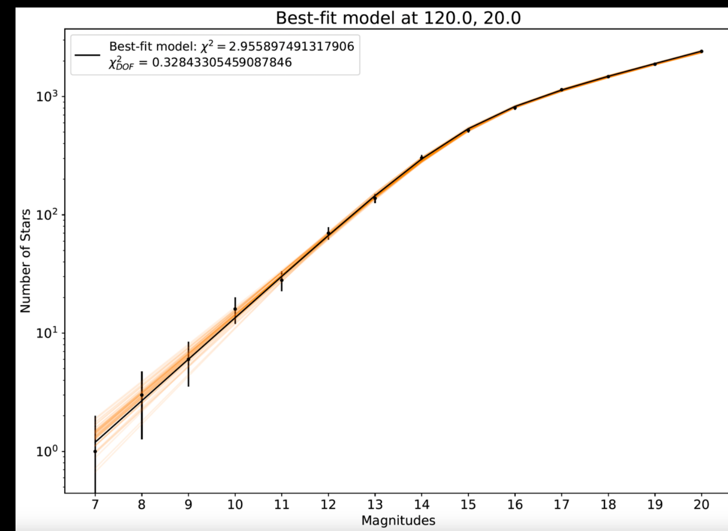


Best-fit model at 180.0, 0.0
Best-fit model: $\chi^2 = 25.398354378320466$
χ^2_{DOF} = 2.8220393753689406
$\ell = 180°, b = 0°$
Σ_b~18,500 deg^2, α_1~0.65,
α_1~0.00,V_b~16.30
Number of Stars
Magnitudes
Best-fit model at 90.0, 20.0
Best-fit model: $\chi^2 = 18.635373012840073$
χ^2_{DOF} = 2.070597001426675
$\ell = 180°, b = 20°$
Σ_b~5700 deg^2, α_1~0.80,
α_1~0.01,V_b~11.45
Number of Stars
Magnitudes
Best-fit model at 90.0, 40.0
Best-fit model: $\chi^2 = 8.394337803857514$
χ^2_{DOF} = 0.9327042004286127
$\ell = 180°, b = 40°$
Σ_b~2900 deg^2, α_1~0.55,
α_1~0.01,V_b~11.69
Number of Stars
Magnitudes
Best-fit model at 180.0, -90.0
Best-fit model: $\chi^2 = 18.181317285440922$
χ^2_{DOF} = 2.5973310407772745
b = −90°
Σ_b~80 deg^2, α_1~0.27,
α_1~0.14,V_b~14.55
Number of Stars
Magnitudes

Number of reference stars

- Can use this form to estimate the effective number of reference stars in any given line-of-sight.

$$\Sigma(V) = \Sigma_b\left[10^{-s\alpha_1(V-V_b)} + 10^{-s\alpha_2(V-V_b)}\right]^{-1/s}$$



- If α_1 =0.4, α_1~0, and $s \gg 1$, the there is an equal contribution per mag for $V < V_b$, and the effective number of reference stars is $N_{\mathrm{ref,eff}} = \Sigma_b\ V_b$
- For $\mathrm{b}{\sim}20°$, α_1~0.4, α_2~0.0, V_b~15, $\Sigma_b{\sim}10^3\mathrm{deg}^{-2}\mathrm{mag}^{-1}$, so $N_{\mathrm{ref,eff}} \sim 10^4\ \mathrm{deg}^{-2}$
- Number of effective reference stars will be much lower at higher Galactic latitudes
- Likely need a FOV of $\sim 10' \times 10'$ to do Earths around the high-latitude starget stars
- $\sim 2\times 5$ times larger than nominal imaging camera.
- Requires roughly 64 8k by 8k CMOS detectors for Nyquist sampling (!)

Scaling with size of the detector

$$S/N \sim f \alpha \Gamma_{\gamma,0}^{1/2} N_{\mathrm{ref,eff}}^{1/2} 10^{-0.2V_b} \ell_d t_{exp}^{1/2} N_d^{1/2} f_{\#}^{-1} (\lambda/D)^{-1}$$

- α is the magnitude of the astrometric signal
- $\Gamma_{\gamma,0}$ is the photon rate per area per time a V=0
- $N_{\mathrm{ref,eff}}$ is the effective number of reference stars
- V_b is the break magnitude
- $t_{exp}^{1/2}$ is the exposure time
- $N_d^{1/2}$ is the number of data points
- D is the diameter
- λ is the wavelength
- ℓ_d is the linear size of the detector $= (\mathrm{Area})^{1/2}$
- $f_{\#}$ is the f-number of the system
- f is a dimensionless constant of order unity, that depends on the Keplerian parameters, shape of the PSF, relation between the diameter and effective area of the telescope

Scaling with size of the detector

$$S/_N \sim f\alpha\Gamma_{\gamma,0}^{1/2} N_{\text{ref,eff}}^{1/2} 10^{-0.2V_b} N_{pix}^{1/2} t_{exp}^{1/2} N_d^{1/2} D$$

- α is the magnitude of the astrometric signal
- $\Gamma_{\gamma,0}$ is the photon rate per area per time a V=0
- $N_{\text{ref,eff}}$ is the effective number of reference stars
- V_b is the break magnitude
- $t_{exp}^{1/2}$ is the exposure time
- $N_d^{1/2}$ is the number of data points
- D is the diameter
- N_{pix} is the number of pixels
- f is a dimensionless constant of order unity that depends on the Keplerian parameters, relation between the effective area of the telescope and the diameter.
- The wavelength cancels out: smaller wavelength→smaller PSF → smaller pixels for Nyquist sampling → small FOV for fixed # of pixels

Astrometric Microlensing

Background stars will be deflected by an amount:

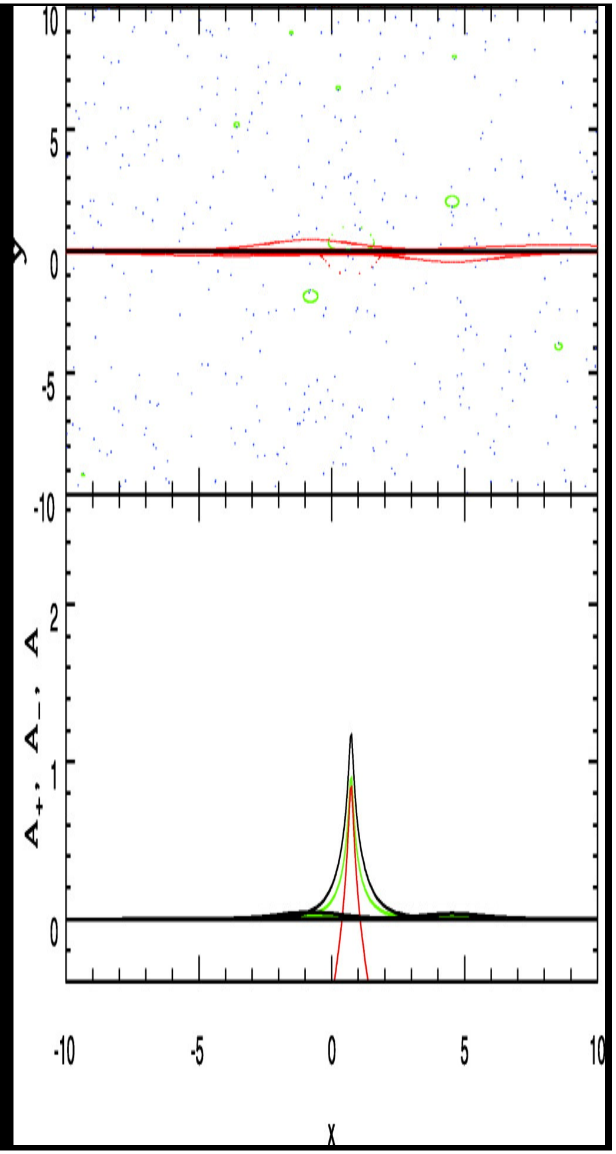
$$\delta(\theta) \sim \frac{\theta_E^2}{\theta}$$

$$\theta_E \approx \sqrt{\kappa M \pi} \sim 0.03'' \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{\pi}{0.1''}\right)^{1/2}$$

- where $\kappa = 8.14 \text{ mas } M_\odot^{-1}$
- The typical angular distance to a reference star is $\sim\theta_d/\sqrt{2}$, where $\theta_d \sim (\text{FOV})^{1/2} \sim 150''$
- The typical size of the astrometric shift is $\sim 8\mu$as.
- Thus, the S/N for all reference stars and all measurements is

$$S/N \simeq 800 \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{\pi}{0.1''}\right)^{1/2} \left(\frac{N_e}{100}\right)^{1/2} \left(\frac{N_{\text{eff,ref}}}{100}\right)^{1/2} \left(\frac{\text{FOV}}{100 \text{ sq.armin}}\right)^{1/2}$$

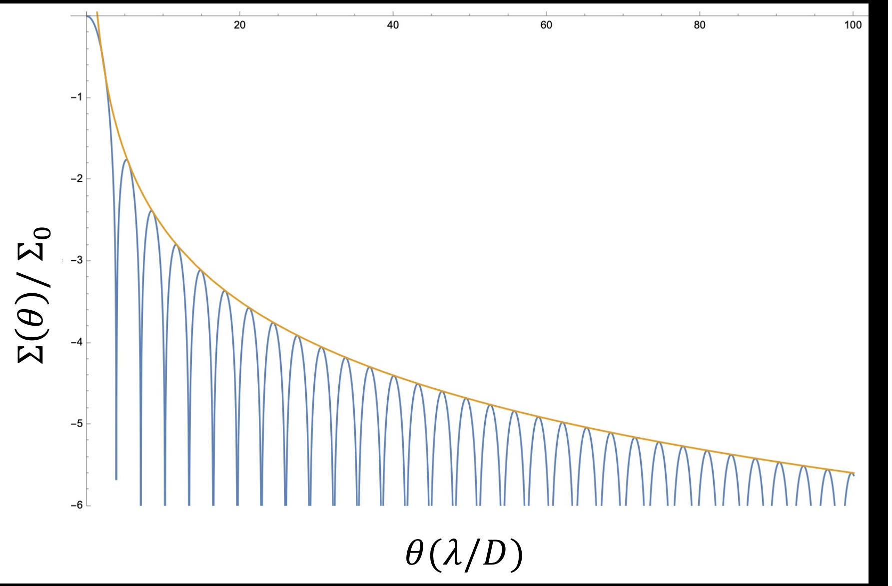
- The uncertainty in the mass is $\frac{\delta M}{M} \sim (S/N)^{-1} \sim 0.1\%$



Background due to halo of the target star

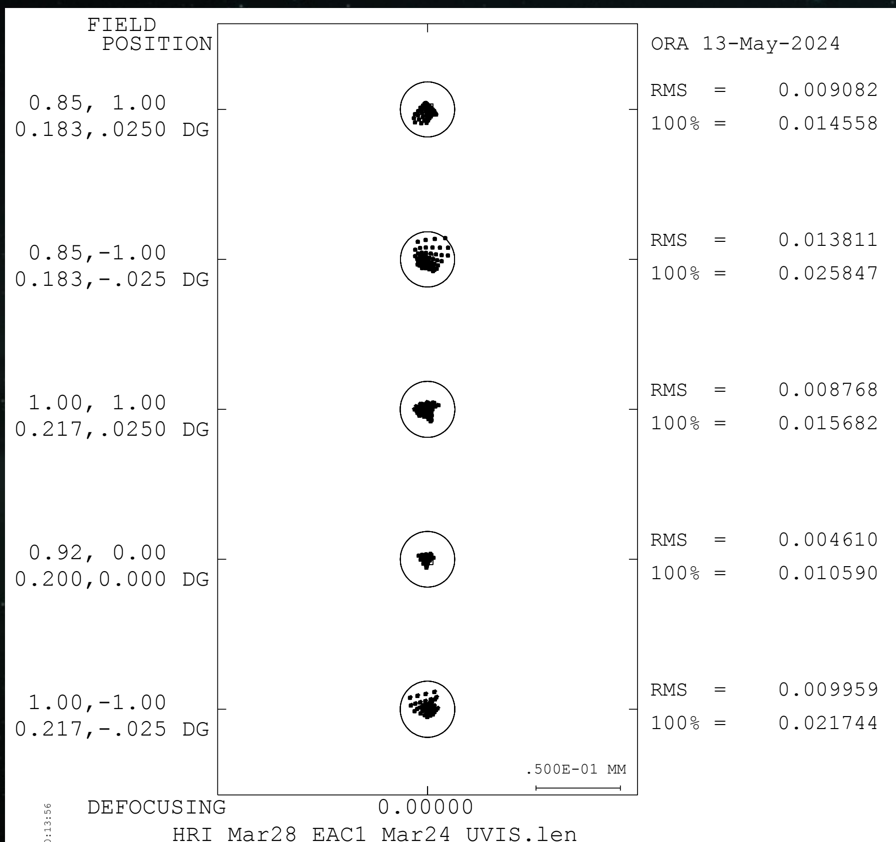
$$\Sigma(\theta) = \frac{1}{2}\Sigma_0 \left(\frac{2J_1[D\theta/\lambda]}{D\theta/\lambda}\right)^2 \sim \frac{2}{\pi^2}\frac{F_*}{(\lambda/D)^2}\left(\frac{D\theta}{\lambda}\right)^{-3}$$

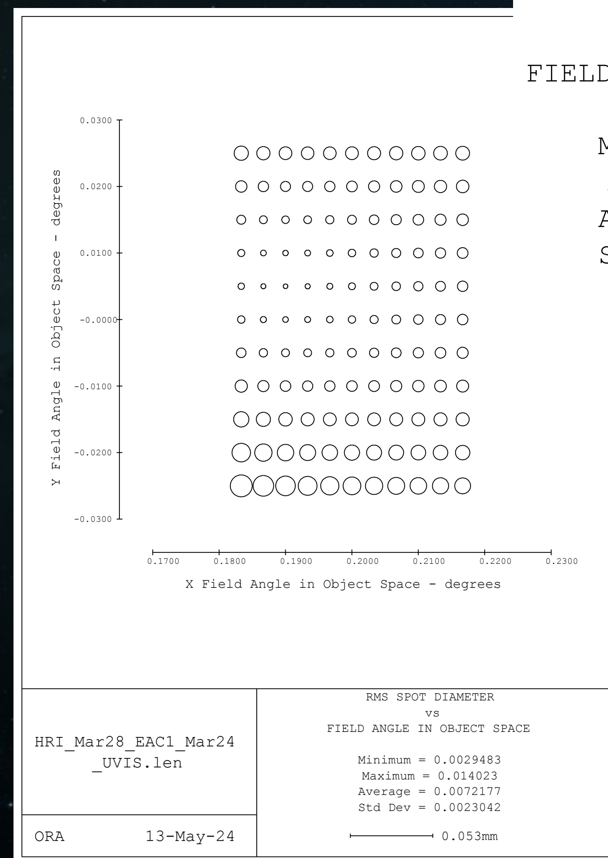
$$F_b = \Sigma\Omega_{\mathrm{PSF}} \sim 10^{-11} F_* \left(\frac{\lambda/D}{0.0159''}\right)^3 \left(\frac{\theta}{1'}\right)^{-3}$$



- At 1’, the flux from the halo of the target star in the PSF of the reference stars is 27 magnitudes fainter!

EAC1 HRI UVIS Spot Sizes

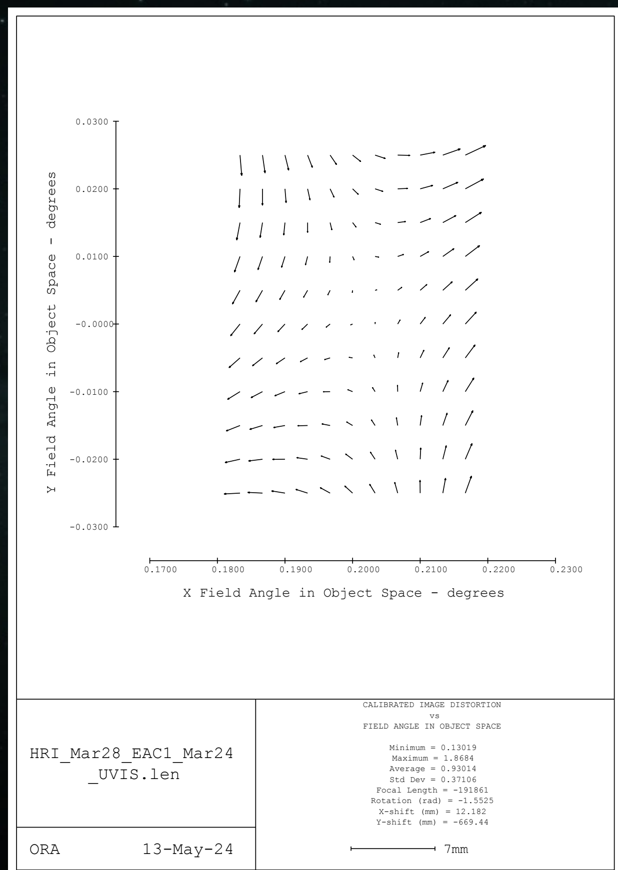




RMS SPOT DIAMETER
vs
FIELD ANGLE IN OBJECT SPACE

Minimum = 0.0029483
Maximum = 0.014023
Average = 0.0072177
Std Dev = 0.0023042

EAC1 HRI UVIS Distortion



CALIBRATED IMAGE DISTORTION
vs
FIELD ANGLE IN OBJECT SPACE

Minimum = 0.13019
Maximum = 1.8684
Average = 0.93014
Std Dev = 0.37106
Focal Length = -191861
Rotation (rad) = -1.5525
X-shift (mm) = 12.182
Y-shift (mm) = -669.44

Plate Scale of 6.99 mas/pixel

Questions

- How well do we need to measure the masses of the planets? (Note: We will also likely get a measurement of the mass of the star.)
- How well we know the orbits? Can this reduce the amount of time required to get masses?
- What wavelengths are best? Are broader or narrower filters better?
- Can we use the guide camera?
- How do we deal with the dynamic range issue?
- How can we control systematics at the $\sim 2\times10^{-6}$ of a pixel level?