Astrometric Explorations of Gravitational Waves

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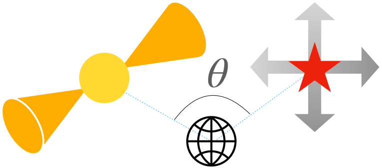
A future space mission with very high precision astrometry

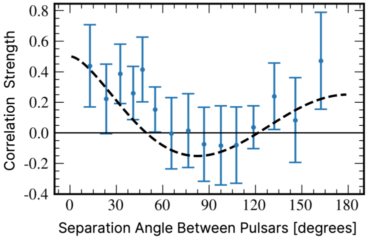
GW Detection with PTA or Astrometry

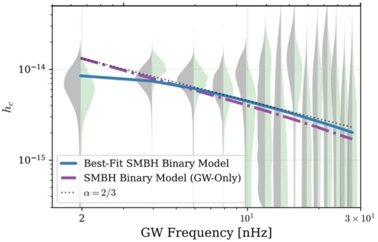
- GW perturbs photon geodesics and the emission/observer point.



- **Galactic GW detectors**:
  - **PTA**: photon frequency shift from $\mathcal{O}(100)$ milli-second pulsars.
  - **Astrometry**: proper motion of $\mathcal{O}(10^9)$ stars/AGNs on celestial plane.

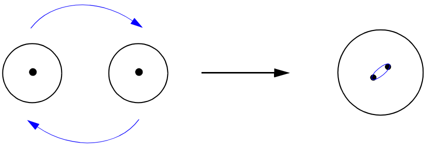
- **PTA** results consistent with quadrupolar correlations and supermassive black hole population [NANOGrav 23']:



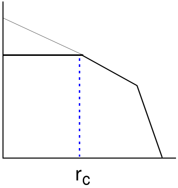


Galaxy Tomography with SGWB Spectrum

Stars and dark matter cause low-frequency turnover through **3-body ejections**.



[Merritt, Milosavljević 02']



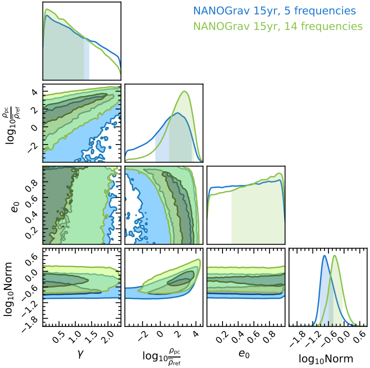
- **Galactic center distribution** prior to disruption:

  $$\rho(r) = \rho_{\rm pc}\left(\frac{r}{1\ \rm pc}\right)^{-\gamma};$$

- 3-**body scattering is favored** with $\rho_{\rm pc} = 10^{(5.5^{+1.7}_{-2.5})}\ M_\odot/\mathrm{pc}^3$.

  [preliminary, Xue, YC et al]

- Precise spectrum measurements and EM observation for individual can help.



[preliminary, Xue, YC et al]

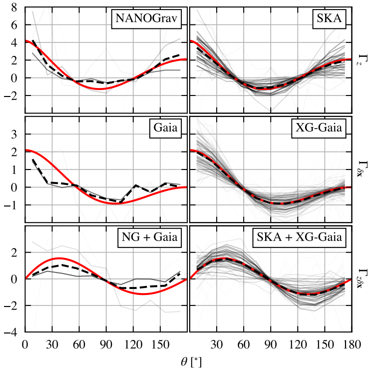
GW Variance and Parameter Estimation

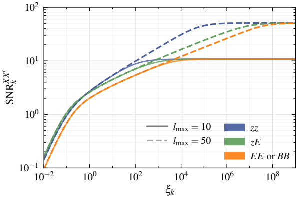
- **Stochastic GW** challenges parameter estimation.
  → GW variance:
  $\sigma^2(\mathcal{P}_P) \propto \langle hhhh \rangle \propto \mathcal{P}_P^2$.

- Variance in configuration space (θ) is **correlated**.
  [Allen 22']

- **Harmonic space**:
  $X_{\ell m} \in (z_{\ell m}, E_{\ell m}, B_{\ell m})$:

- SNR **saturates** at strong signal, $\ell_{\max} \sim \sqrt{N_X/2}$.
  Astrometry with $N_X \sim 10^9$ can help.





[Çalışkan, YC, Dai, Kumar, Stomberg, Xue, JCAP 2312.03069]

Amplitude and Spectral Index

Power-law SGWB: $h_c \equiv A \left(\frac{f}{f_{\rm ref}}\right)^{\alpha}$.

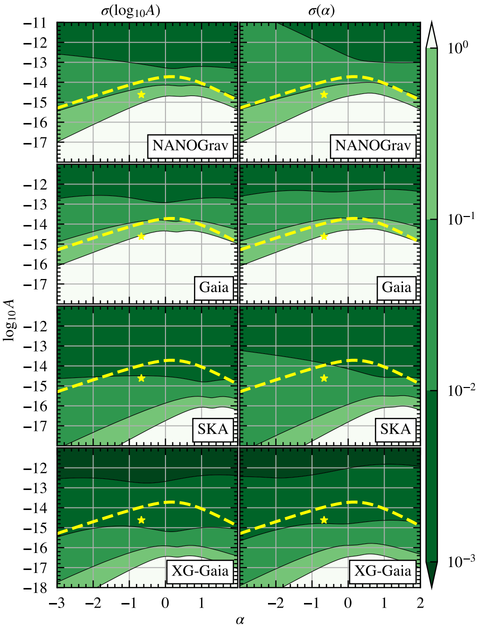
- **Fiducial**: $A \sim 10^{-15}$, $\alpha \sim -\frac{2}{3}$ is consistent with SMBHB.
- **4 examples**:

  Current: **NANOGrav, Gaia**,

  Future: **SKA, XG-Gaia**.
- XG-observations:

  SKA and **XG-Gaia**

  measures spectrum precisely.

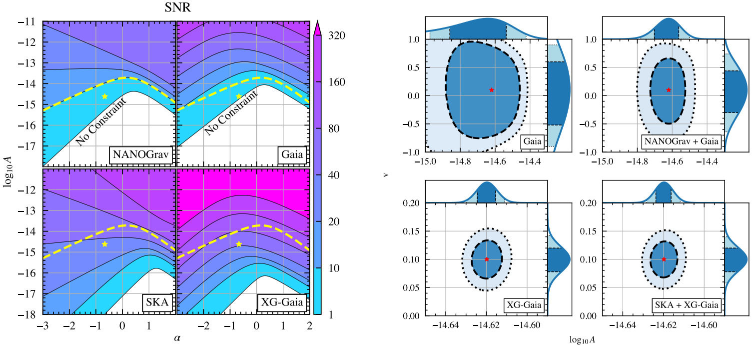


[Çalışkan, YC, Dai, Kumar, Stomberg, Xue, JCAP 2312.03069]

Total SNR and Circular Polarization

- zz, EE, BB and zE pairs are sensitive to total intensity $I \equiv \langle h_L h_L + h_R h_R \rangle$.

  Gaia observation can marginally cross-check PTA results.



[Çalışkan, YC, Dai, Kumar, Stomberg, Xue, JCAP 2312.03069]

- EB and zB are sensitive to circular polarization $V \equiv \langle h_L h_L - h_R h_R \rangle$.

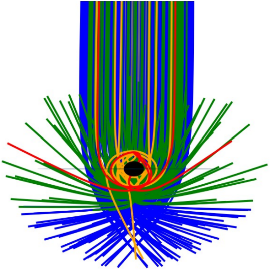
  Synergistic observation with NANOGrav-Gaia can resolve circular polarization fraction V/I better than **Gaia**-only observation.

Photon Ring Astrometry for Metric Perturbations

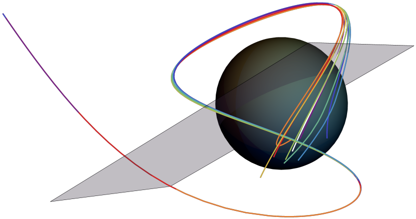
Event Horizon Telescope: best-ever angular resolution from VLBI.

Future: **ngEHT** and space-VLBI **BHEX**.

Photon orbits [KGEO]



[YC, Xue, Brito, Cardoso, PRL 2211.03794]



Photon bound orbits outside BHs:

Photon ring with enhanced intensity.

$\rightarrow$ GR and BH test.

[Fundamental Physics Opportunities with the ngEHT, Ayzenberg et al, 2312.02130]

Astrometry for gravitational waves?

Geodesics deviations **grow exponentially** near critical orbits.

- **Ultralight boson clouds** [YC, Xue, Brito, Cardoso, PRL 2211.03794].
- **Ringdown tomography** [Zhen, Cardoso, YC, 2408.10303].

Summary

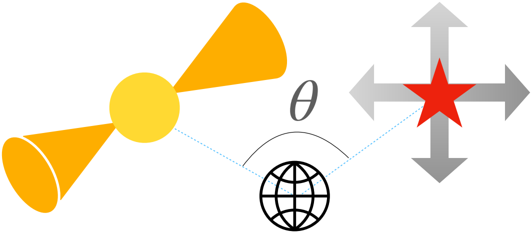
- SGWB at nHz have been confirmed by **PTA** through spatial correlations.

  Potential turning at low frequency: galaxy tomogrphy?

- GW variance challenges parameter estimation, easier in harmonic space.

- **Astrometry** can help with 10^9 stars and polarization sensitivity.

- **Photon ring astrometry**: boost deviation via orbit instability near BH.

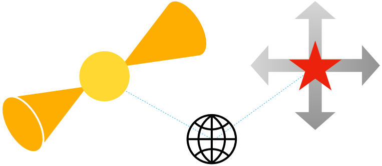


Thank you!

Backup

GW Detection with PTA or Astrometry

- GW perturbs photon geodesics and the emission/observer point.



- **Galactic GW detectors**:

  • **PTA**: photon frequency shift from $\mathcal{O}(100)$ milli-second pulsars.

  Redshift $\delta z \propto n^i n^j h_{ij}(E) + \cdots h_{ij}(P)$.

  Pulsar term $\sim h_{ij}(P)$ is usually **neglected** due to distance uncertainty and incoherent among different pulsars.

  Timing residue $\sim \int \delta z \mathrm{d}t$: sensitivity decreases at higher frequency.

  • **Astrometry**: proper motion of $\mathcal{O}(10^9)$ stars/AGNs on celestial plane.

  Angular deflection $\delta n^i = R^{ijk} h_{jK}(E)$ [Book, Flanagan 10'].

- **Distinct response functions** between PTA ($R^{ij} \propto n^i n^j$) and astrometry (R^{ijk}) show complimentary sensitivity to GW polarization.

Stochastic Gravitational Wave Background

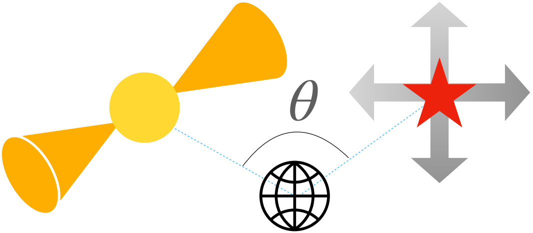
- Stochastic GW background: incoherent sum of GWs at each frequency.
- Two-point correlations using **power spectral density** $\mathcal{P}_P(f, \hat{\mathbf{\Omega}})$:

$$\langle h_P(f, \hat{\mathbf{\Omega}})\, h_{P'}(f', \hat{\mathbf{\Omega}}')^* \rangle = \delta(f - f')\, \delta(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}')\, \delta_{PP'} \mathcal{P}_P(f, \hat{\mathbf{\Omega}}), \;\; P \in (L/R).$$

- N_X observables $X \sim R_X^{ij} h_{ij} \to N_X(N_X - 1)/2$ pairs : $\langle XX' \rangle \propto \mathcal{P}_P \Gamma_{XX'}$.
- Isotropic GW $\to$ generalized Hellings-Downs curves [HD 83', Mihaylov et al 18']:

$$\Gamma_{XX'} \propto \int R_X^{ij} \epsilon_{ij} R_{X'}^{lk} \epsilon_{lk} \mathrm{d}\mathbf{\Omega}, \qquad \text{GW polarization basis } \epsilon_{ij}.$$

- **Microscopic quadrupolar tensor nature** emerges at macroscopic scales.



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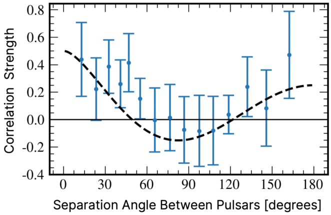
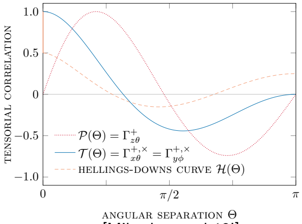


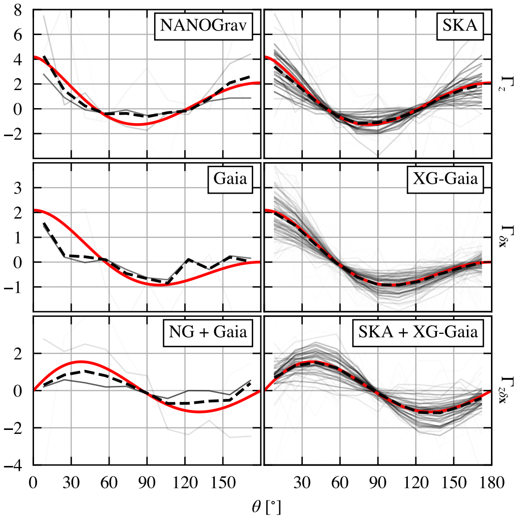
[NANOGrav 23']



[Mihaylov et al 18']

Generalized Hellings Downs Curves with Cosmic Variance

- **Gaussian SGWB** $\mathcal{P}_P \propto \langle hh \rangle$ $\rightarrow$ GW variance: $\sigma(\mathcal{P}_P) \propto \langle hhhh \rangle \propto \mathcal{P}_P$.
- Total uncertainty = noise + total variance + cosmic variance [Allen 22']
- Cosmic variance persists in the limit $N_X \rightarrow \infty$.
- Variance in configuration space (θ) is **correlated**.



[Çalışkan, YC, Dai, Kumar, Stomberg, Xue, 2312.03069]

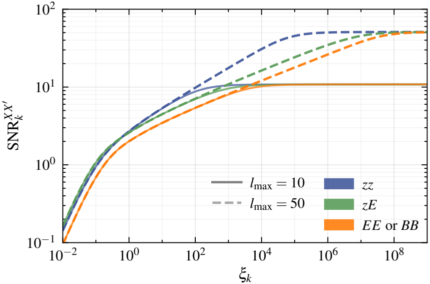
Harmonic Space Estimators

- **Spherical harmonic expansion** [Gair et al 14', Qin et al 18']:

$$\delta z_a = z_{\ell m} Y_{\ell m}(n_a), \qquad \delta n_a = E_{\ell m} \mathbf{Y}^E_{\ell m}(n_a) + B_{\ell m} \mathbf{Y}^B_{\ell m}(n_a), \qquad \ell \geq 2.$$

- **Diagonalization in harmonic space** $X_{\ell m} \in (z_{\ell m}, E_{\ell m}, B_{\ell m})$:

$$\langle X_{\ell m} X'_{\ell' m'} \rangle \propto C^{\ell}_{XX'} \delta_{\ell \ell'} \delta_{mm'}, \qquad \sigma(X_{\ell m} X'_{\ell m}) \propto |C^{\ell}_{XX'}|^2 + C^{\ell}_{XX} C^{\ell m}_{X'X'}.$$

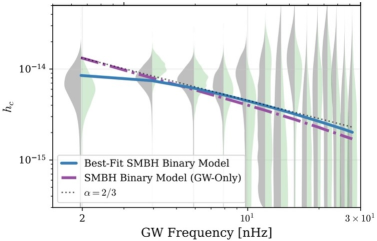


- SNR **saturates** at strong signal region, dependent on $\ell_{\max} \sim \sqrt{N_X/2}$.

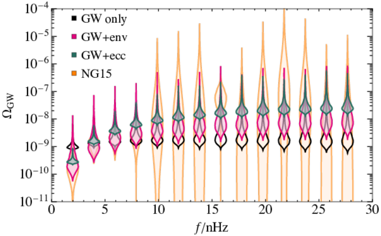
Astrometry with $N_X \sim 10^9$ can help. However, $C^{\ell}_{EE/BB} \propto 1/\ell^6$ and $C^{\ell}_{zz} \propto 1/\ell^4$.

Low-frequency Turnover in SGWB Spectrum

- **Circular orbit** driven by GW emission: $h_c \propto f^{-2/3}$.
- **NANOGrav** data slightly prefers a spectrum turnover at $f_{\rm min} = 2\,\mathrm{nHz}$:



[NANOGrav 23']

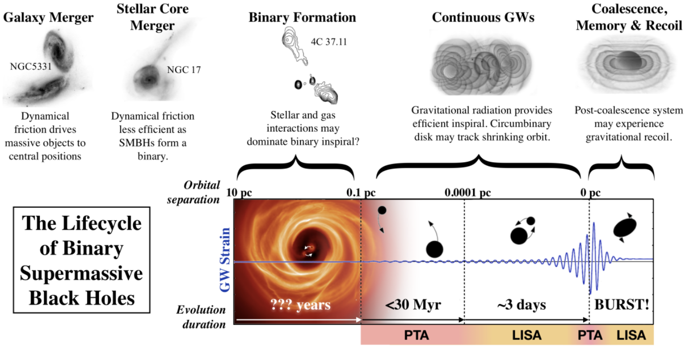


[Raidal et al 24']

- Hardening process or highly eccentric orbits?
- As $f_{\rm min} \sim 1/T$ and $T = 15\,\mathrm{yrs}$, **longer observation time** can confirm.

Orbital Hardening with Stars or Dark Matter

- **SMBHB formation**:

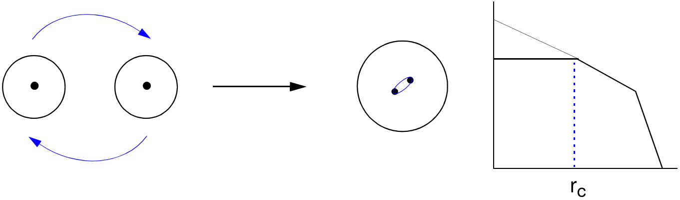


[Sarah Spolaor NANOGrav]

- Final parsec problem: 3-body ejection of stars/dark matter.
- Dominant orbital hardening rate between dynamic friction (early) and GW emission (late).

Three-body Scattering vs N-body Simulation

- Hardening rate from 3-body scattering experiment [Quinlan 96']: $\frac{\mathrm{d}a}{\mathrm{d}t} \propto \frac{G\rho}{\sigma} a^2$; and eccentricity growth $\frac{\mathrm{d}e}{\mathrm{d}t} > 0$.
- More efficient than 2-body dynamical friction within the hardening radius $a_h \equiv r_i q/(4(1+q)^2)$ where $\int_0^{r_i} \rho \, \mathrm{d}^3x = M_{\mathrm{BH}}$.
- **Strong backreaction**:
  N-body simulation: 2 dressed clumps $\rightarrow$ 1 flat core [Milosavljević, Merritt 01']:

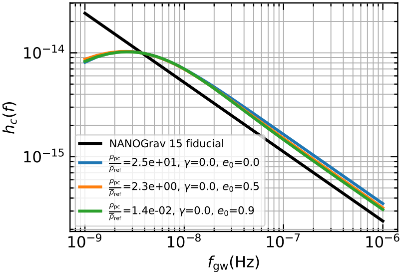


[Merritt, Milosavljević 02']

- Orbital evolution of the two methods **match** by taking ρ/σ at r_i [Sesana, Khan 15'].

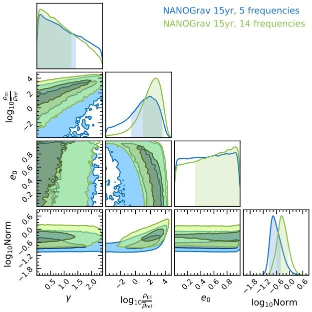
Galactic Center Distribution and SGWB Spectrum

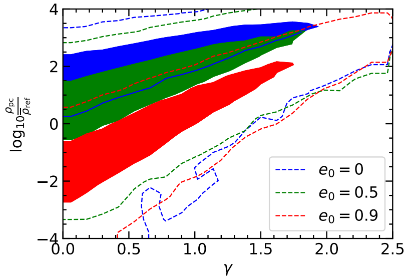
- **Galactic center distribution** prior to disruption: $\rho(r) = \rho_{\rm pc}\left(\frac{r}{1\,{\rm pc}}\right)^{-\gamma}$; e.g., $\gamma = 0$ (core), 1 (NFW), 7/3 (spike).
- Each $(\rho_{\rm pc}, \gamma)$ and $(M_{\rm BH}, q)$ determines r_i where binary is formed with e_0.
- SGWB prediction: evolving ${\rm d}e/{\rm d}a$ for each $(M_{\rm BH}, q, \rho_{\rm pc}, \gamma, e_0)$ from r_i, and integrated over SMBHB population $(M_{\rm BH}, q, z)$ from `holodeck`.
- Gaussian prior on **normalization** of the population distrbution.
- Best-fit spectrum for various e_0:



[preliminary]

Data Analysis





[preliminary]

- 3-**body scattering is necessary** with $\rho_{\rm pc} = 10^{(5.5^{+1.7}_{-2.5})}\ M_\odot/{\rm pc}^3$, unless $e_0(r_i) > 0.999$.
- Degeneracy between $\rho_{\rm pc}$ and γ follows constant ρ_i/σ_i.
  Low γ is favored.
- Degeneracy between $\rho_{\rm pc}$ and e_0: resolving coherent individual or checking correlations among different frequencies [Raidal et al 24'] ?

Summary

- Nano-hertz SGWB have been confirmed by **PTA** through spatial correlations.
- The SGWB spectrum is consistent with **SMBHB**.
- **Potential turnover** at low frequency:
  - **3-body ejection** of galactic-center stars or dark matter;
  - High eccentricity?
- **Future**:
  - breaking degeneracy of $\rho_{\rm pc}$ and e_0.
  - distinguishing stars or dark matter?

Evolution of Eccentric Orbits

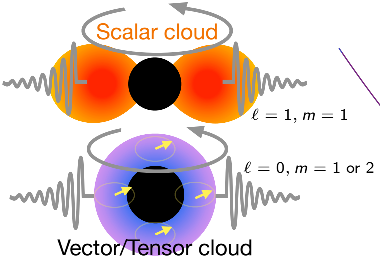
$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{64}{5}\frac{G^3M^3}{c^5a^3}\frac{q}{(1+q)^2}\frac{(1+\frac{73}{24}e^2+\frac{37}{96}e^4)}{(1-e^2)^{7/2}} - HG\frac{\rho_i}{\sigma_i}a^2,$$

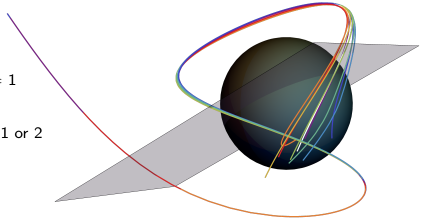
$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{304}{15}\frac{G^3M^3}{c^5a^4}\frac{q}{(1+q)^2}\frac{e(1+\frac{121}{304}e^2)}{(1-e^2)^{5/2}} + HK(e,a)G\frac{\rho_i}{\sigma_i}a.$$

$$H \approx 18, \qquad K \approx 0.3e\left(1-e^2\right)^{0.6}\left(1+\frac{a}{0.2\,a_h}\right)^{-1}$$

Photon Ring Astrometry for Gravitational Atoms

- Superradiant clouds generate local oscillatory metric perturbations $g_{\mu\nu} \simeq g_{\mu\nu}^{\mathrm{K}} + \epsilon h_{\mu\nu}$ that **deflect geodesics** $x^\mu \simeq x^\mu_{(0)} + \epsilon x^\mu_{(1)}$:





[YC, Xue, Brito, Cardoso, PRL. **130** (2023) no.11, 111401]

- **Axion/scalar cloud** mainly causes time delay [Khmelnitsky, Rubakov 13'].

- **Polarized vector or tensor cloud** contribute to both time delay and **spatial deflection**.

- Photon ring autocorrelations [Hadar et al 20'] **probe** $M_{\mathrm{cloud}}/M_{\mathrm{BH}}$ **to** 10^{-3} **for vector and** 10^{-7} **for tensor**.